# Quantum and statistical fluctuations in dynamical symmetry breaking \*

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#### Abstract

Dynamical symmetry breaking in an expanding nuclear system is investigated in semi-classical and quantum framework by employing a collective transport model which is constructed to mimic the collective behavior of expanding systems. It is shown that the fluctuations in collective coordinates during the expansion are developed mainly by the enhancement of the initial fluctuations by the driving force, and that statistical and quantum fluctuations have similar consequences. It is pointed out that the quantal fluctuations may play an important role in the development of instabilities by reducing the time needed to break the symmetry, and the possible role of quantal fluctuations in spinodal decomposition of nuclei is discussed.

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#### 1 Introduction

Dynamical symmetry breaking is a general behavior in nature and is observed in many field of physics. For example, phase transitions are often related to a dynamical symmetry breaking. The liquid-gas phase transition in infinite matter may be viewed as breaking of translational invariance which is related to mechanical instability of the system against small density fluctuations inside the spinodal zone. The spinodal instability may provide a possible mechanism for triggering the fragmentation of a hot piece of nuclear matter produced in heavy-ion collisions [1, 2]. The composite system formed in the overlap zone of the colliding ions expands and cools down. Depending on the initial compression and temperature, it may enter the unstable region of the phase diagram. In such a scenario, the fragment formation takes place by a rapid growth of density fluctuations in the spinodal region. Such a scenario is valid for infinite matter or a finite system. In matter, unstable modes are sound modes characterized by plane waves, whereas in a finite system these are collective vibrations associated with density fluctuations with multipolarity L and M. For example, a spherical system can break into pieces as soon as the collective mode with multipolarity L larger than one becomes unstable.

Stochastic one-body transport models provides a suitable framework for a theoretical description of the collision process all the way from the initial stage up to the final state involving development of large density fluctuations and formation of fragments [3, 4, 5]. This approach in a semi-classical approximation for the phase-space density, which is frequently referred as the Boltzmann-Langevin model, has been applied to investigate the spinodal decomposition of nuclear systems [6]. Stochastic Time Dependent Hartree Fock (STDHF) approaches are now under development in order to take into account the quantal nature of single particle motion [7, 8].

A valuable insight on dynamical symmetry breaking, and the role of quantal and statistical fluctuations on the instabilities can be gained by investigating the early development of density fluctuations. The linear response treatment of the one-body stochastic transport models, in semi-classical or quantal framework, provides a useful basis for this purpose. In a recent work [9], the dominant collective modes in an expanding finite nuclear system are identified in a linear response treatment by solving temperature dependent quantal RPA equation, and the properties of these collective modes are investigated as a function of dilution of the system. These calculations

illustrate that, as the system expands, the collective modes which were all stable at normal density, become softer until their energies become zero and then bifurcate toward the imaginary values, signaling the instability of the corresponding mode.

The density fluctuations associated with the collective degrees of freedom can be determined in terms of collective transport equations. These transport equations for the collective variables can be deduced from the microscopic transport models by projecting the equation of motion onto a specific collective mode [10, 11], or can be derived by some other means [12]. Since, the standard one-body transport approach provides a classical description of the collective motion, the deduced collective models are best suited at high excitation energies when the temperature is much larger than the typical frequencies of the collective motion. On the other hand, when the characteristic energies of the collective modes are comparable with or larger than temperature of the system, the quantal fluctuations of the collective dynamics become important or even dominant. Then, a quantum transport approach of the collective variables is required for describing dynamics of large density fluctuations. The stochastic one-body transport description can be improved by incorporating the memory effects associated with finite duration of binary collisions. As demonstrated in a recent work [13, 14], in such non-Markovian description, transport properties of collective motion evolve in accordance with the quantal fluctuation-dissipation relation [15].

In this work, we consider a single collective mode, specifically the quadrupole moment of the fluctuating density in an expanding nuclear system and study its evolution by employing collective transport models, in both classical and quantal forms. Although these models can be derived from the underlying microscopic transport models in Markovian and non-Markovian forms, we follow a phenomenological approach and parameterize the basic ingredients of the model in accordance with the recent TDHF calculations for expansion of a hot spherical system [16, 17], and the RPA calculations for describing the unstable collective modes [9]. In the Fokker-Planck approach, the coupling between the collective mode and the intrinsic degrees of freedom is described in terms of friction and diffusion coefficients. This simple but realistic model allows us to calculate the development of fluctuations associated with the collective mode during the expansion of the nuclear system, and to investigate the role of quantal and statistical fluctuations in the dynamics of the symmetry breaking.

#### 2 The model

#### 2.1 The collective Hamiltonian

Combining the results of the finite temperature TDHF description of the expansion of hot nuclear sources ref. [16, 17] and the RPA investigation of the stability of a finite hot expanding nuclear source [9], we can determine the most important collective modes and describe their transitions from stable to unstable configurations as a function of expansion, temperature and source size. Here, we specifically consider evolution of the quadrupole shape fluctuations in an expanding spherical hot nuclear source. According to finite temperature TDHF calculations with initial temperatures in the range of T = 5 - 10 MeV, the system executes monopole vibrations and cools down by particle evaporation. Depending on the size of the source and the initial temperature, it takes between 30 fm/c and 75 fm/c to reach the maximum dilution that occurs typically around 1/3 of the normal nuclear matter density. According to the constraint RPA calculations, the quadrupole mode with an initial frequency  $\omega$ , which is found to be about  $\omega \simeq 65A^{-1/3}$  MeV  $\hbar$ at moderate temperature, becomes softer as the system expands. It eventually becomes unstable and reaches an imaginary frequency of about  $i20A^{-1/3}$  $\text{MeV}/\hbar$  at the maximum dilution.

In order to describe the evolution of the quadrupole mode during the expansion of the source, we consider the following model, in which the Hamiltonian of the collective motion is taken as

$$H = \frac{P^2}{2M} + U(Q) \tag{1}$$

where  $M=3AmR_0/8\pi$  is the irrotational mass parameter associated with the collective variable Q determined by the sharp nuclear surface  $R_0$ , A is the number of nucleons in the source and U(Q) represents the collective potential energy. For small deformation Q, the potential energy has a harmonic form  $U(Q)=K(t)\,Q^2/2+\ldots$  with a time dependent spring constant  $K(t)=K_0g(t)$  where  $K_0=\omega^2M$  and  $\omega=65A^{-1/3}$  MeV/ $\hbar$  denotes the frequency of the giant quadrupole vibrations at normal density. The time dependent factor g(t) in the spring constant describes the transition from stable to unstable regime during the expansion of the source. This factor is parameterized In accordance with the TDHF and RPA calculation: starting from g(0)=1 it monotonically decreases in time and reaches a value around

 $g(t_0) = -1/9$  at a characteristic time  $t_0$  for the source to reach the maximum instability (typically,  $t_0 = 50$  fm/c). The harmonic approximation is valid during early stages of the unstable evolution when the magnitude of fluctuations is small. In order to describe large fluctuations, we need to incorporate an anharmonic terms in the potential energy, U(Q), which prevent the fluctuation to grow indefinitely without bound. For symmetry considerations, we add an anharmonic term of the form  $Q^4$  in the potential energy to achieve the saturation of the fluctuations

$$U(Q) = \frac{K(t)}{2}Q^2 + \beta Q^4 \tag{2}$$

Since, we are mainly concern with the early development of fluctuations, the precise form of this non-linear term is not important for this purpose. The numerical coefficient in front of the non-linear term is taken to be  $\beta = K_0/2$  in order to obtain saturation of fluctuations at a reasonable value of the collective variable Q which corresponds to the quadrupole deformation of the fragmenting system (e.g. the quadrupole deformation may lead to break-up of a system into two pieces). The numerical calculations presented are performed for a system containing A = 64 nucleons.

# 2.2 Classical Langevin dynamics

The classical Langevin approach provides a useful framework for describing the dynamics of the collective motion including dissipation and fluctuations. In this approach, the trajectory in the collective phase space,  $(Q^{(i)}(t), P^{(i)}(t))$  of an event i, is determined by solving a Langevin equation,

$$\frac{dQ^{(i)}}{dt} = \frac{P^{(i)}}{M} \tag{3}$$

$$\frac{dP^{(i)}}{dt} = \frac{\partial U\left(Q^{(i)}\right)}{\partial Q} - \gamma P^{(i)} + \xi^{(i)}\left(t\right) \tag{4}$$

where  $\gamma$  is the friction coefficient and  $\xi^{(i)}(t)$  is the random force arising from the coupling of the collective degrees of freedom with the single particle motion. The fluctuating force  $\xi^{(i)}(t)$  is assumed to be a white noise, and specified by a Gaussian distribution with zero mean  $\langle \xi(t) \rangle = 0$  and a second moment

$$\langle \xi(t) \xi(t') \rangle = \delta(t - t') 2D \tag{5}$$

where  $\langle ... \rangle$  represents the statistical average over the ensemble of events i. The diffusion coefficient D is determined in terms of the friction coefficient by the classical dissipation-fluctuation relation

$$D = \gamma MT \tag{6}$$

where T represents the time-dependent temperature of the internal system which may be specified by assuming an isentropic expansion of the nuclear source. The only remaining parameter of the model is the friction coefficient, which may be estimated from the observed damping width of the giant quadrupole excitation according to  $\gamma = \Gamma/\hbar$ . In the calculations, we take the damping width to be  $\Gamma = 85A^{-2/3}$  MeV.

By generating a sufficiently large number of events, we can construct the phase-space density f(P,Q,t) associated with the collective variables P and Q as

$$f(Q, P, t) = \frac{1}{N} \sum_{i=1}^{N} \delta(Q - Q^{(i)}(t)) \delta(P - P^{(i)}(t))$$
 (7)

where N denotes the number of events. The probability distribution n(Q, t) of Q and the variances of the collective variables  $\sigma_Q(t)$  and  $\sigma_P(t)$  are calculated as

$$n(Q,t) = \int dP \ f(Q,P,t) = \frac{1}{N} \sum_{i=1}^{N} \delta\left(Q - Q^{(\lambda)}(t)\right) \tag{8}$$

and

$$\sigma_Q(t) = \langle Q^2 \rangle = \int dP dQ \ Q^2 f(Q, P, t) \tag{9}$$

$$\sigma_P(t) = \langle P^2 \rangle = \int dP dQ \, P^2 f(Q, P, t) \tag{10}$$

The initial conditions of the events generated by the Langevin equation is specified by the Boltzmann distribution with the initial temperature  $T_0$ 

$$f_0(Q, P) = \frac{1}{Z} \exp\left(-\frac{H(Q, P)}{T_0}\right). \tag{11}$$

In our simulations, we sample the initial fluctuations using a metropolis algorithm[18].

The classical description of the Langevin equation provides a good approximation at sufficiently high temperatures at which the dynamics is dominated by thermal fluctuations. At low temperatures, the Langevin approach

may be improved by incorporating the quantal fluctuations through the initial conditions and by specifying the diffusion coefficient through the quantal fluctuation-dissipation relation [13, 14]. As a matter of fact, this description produces exact quantal phase-space density in the harmonic limit, for which the quantal fluctuation-dissipation relation may be expressed as  $D = (\gamma M\hbar\omega/2) \coth(\hbar\omega/2T)$  for real  $\omega$ . However, in general for non-linear evolution, collective motion should be investigated in the basis of a quantal transport equation.

#### 2.3 Quantal transport model

In quantal approaches the system is described by its density matrix  $\hat{\rho}(t)$  on the collective Hilbert space,  $\hat{\rho}(Q,Q',t)=\langle Q|\,\hat{\rho}(t)\,|Q'\rangle$ , which is determined by a quantal transport equation

$$i\hbar \frac{\partial}{\partial t}\hat{\rho}(t) = \left[\hat{H}(t), \hat{\rho}(t)\right] + i\hbar \hat{K}(\hat{\rho})$$
 (12)

where  $K(\hat{\rho})$  represents a "collision term" arising from the coupling of the collective motion with the intrinsic variables, which corresponds to the friction and diffusion term in the Langevin equation. The collective density matrix may be expanded in terms of a complete set of time-dependent wave functions  $|\Psi_i(t)>$  as

$$\hat{\rho}(t) = \sum_{i,j} |\Psi_i(t)\rangle \rho_{ij}(t) \langle \Psi_j(t)|$$
(13)

where the time-dependent wave functions are obtained by solving the timedependent Shrödinger equation

$$i\hbar \frac{\partial}{\partial t} |\Psi_i(t)\rangle = \hat{H}(t) |\Psi_i(t)\rangle$$
 (14)

and the elements of density matrix  $\rho_{ij}(t)$  obey a master equation

$$\frac{\partial}{\partial t}\rho_{ij}(t) = \langle \Psi_i(t)|K(\hat{\rho})|\Psi_j(t)\rangle \tag{15}$$

If the system initially is in statistical equilibrium at an initial temperature  $T_0$ , it can be represented by a density matrix,

$$\hat{\rho}(t=0) = \frac{1}{Z} \exp\left(-\frac{\hat{H}_0}{T_0}\right) \tag{16}$$

It is convenient to introduce the eigenstates  $|\varphi_i\rangle$  and the eigen-energies  $E_i$  of the initial Hamiltonian  $H_0 = H(t=0)$ 

$$H_0|\varphi_i\rangle = E_i|\varphi_i\rangle \tag{17}$$

as initial basis. Consequently, the initial density matrix reduces to Boltzmann occupation probabilities of the eigenstates of the initial Hamiltonian,

$$\rho_{ij}(t) = \delta_{ij} \frac{1}{Z} \exp\left(-\frac{E_i}{T_0}\right). \tag{18}$$

The probability density n(Q,t), and the variances  $\sigma_Q(t)$  and  $\sigma_P(t)$  of the collective variables are calculated according to

$$n(Q,t) = \langle Q | \hat{\rho}(t) | Q \rangle \tag{19}$$

and

$$\sigma_Q(t) = \langle \hat{Q}^2 \rangle = \operatorname{tr} \hat{Q}^2 \hat{\rho}(t) \tag{20}$$

$$\sigma_P(t) = \langle \hat{P}^2 \rangle = \operatorname{tr} \hat{P}^2 \hat{\rho}(t) \tag{21}$$

Since the model Hamiltonian is nearly harmonic at the initial time t=0, the initial variances of the collective variables can approximately be expressed in a closed form as

$$\sigma_{Q}(0) = \langle Q^{2} \rangle_{0} \simeq (\hbar \omega / 2K_{0}) \coth(\hbar \omega / 2T_{0}) \xrightarrow{T_{0} \to 0} \hbar \omega / 2K_{0}$$

$$\sigma_{P}(0) = \langle P^{2} \rangle_{0} \simeq (M\hbar \omega / 2) \coth(\hbar \omega / 2T_{0}) \xrightarrow{T_{0} \to 0} M\hbar \omega / 2$$
(22)

Here, the approximate expressions are valid when temperature is small as compared to the collectives frequency,  $T_0 \ll \hbar \omega$ , and hence the initial variances are dominated by the ground state zero point fluctuations. In the opposite limit when temperature is much larger than half the initial collective energy  $\hbar \omega \ll 2T$  the initial variances of the collective variables can be approximated by

$$\sigma_Q(0) \simeq T_0/K_0 
\sigma_P(0) \simeq MT_0$$
(23)

which corresponds to the classical description discussed in the previous section.

### 3 Results

We carry out a number of simulations on the basis of two different approaches presented above to investigate the development of quadrupole shape instabilities in an expanding spherical nuclear system prepared at a range of initial temperatures and calculate the probability density n(Q, t) and the variances  $\sigma_Q(t)$ ,  $\sigma_P(t)$  of the collective variables. In our calculations, we consider that the initial state is prepared at normal density and at a temperature in the range of T=5-10 MeV. This is in accordance with the BUU calculations for central heavy-ion collisions around Fermi energy, which show that the collisions lead to an equilibrated composite spherical system at temperatures around T=10 MeV. The second important parameter in our model is the characteristic time  $t_0$  for expansion until the system reaches the turning point, which has a typical magnitude around  $t_0=50$  fm/c. In order to see the influence of the expansion time on the dynamics of symmetry breaking, we present calculations for different  $t_0$ .

#### 3.1 Classical dynamics

The result of simulations of the classical Langevin equation is illustrated in figure 1 by solid lines, in which the probability distribution n(Q,t) and the potential energy U(Q,t) are plotted as a function of Q at several times for two different initial temperatures  $T_0 = 5 \text{ MeV}$  (left panel) and  $T_0 = 10 \text{ MeV}$ (right panel). In these simulations temperature is assumed to be constant and equal to the initial value. The width of distribution rapidly increase as the potential gets softer and the probability distribution splits into two components after the potential exhibits a local maximum at Q=0. Figure 2 displays the variances associated with the collective variable Q and the collective momentum P as a function of time for the initial temperatures T=5MeV and T=10 MeV. The fluctuation in the collective coordinate rapidly increase until the system reaches the maximum instability and then saturates after 100 fm/c. During the same time the momentum distribution remains almost constant and goes to the asymptotic value given by the equipartition theorem  $\sigma_P \simeq MT_0$ . In figures 1 and 2, the dotted lines display the results of the pure potential calculations without the dissipation and fluctuations. During the expansion phase until the source reaches maximum dilution at about t = 50 - 60 fm/c, the pure potential result for n(Q, t) is very close to the Langevin simulations with friction term and stochastic force. Therefore,

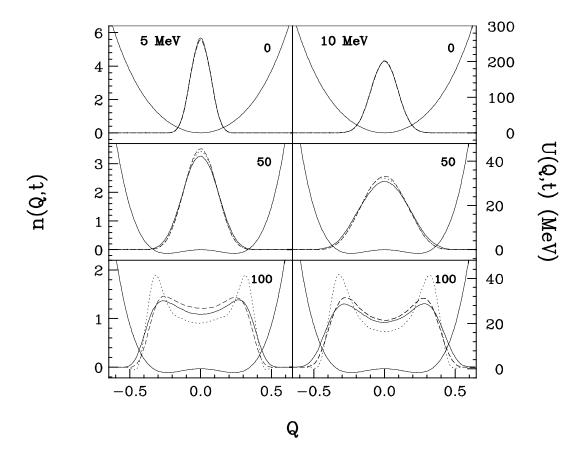


Figure 1: Probability distribution of collective variable and potential energy as a function of Q at times t=0,50,100 fm/c for initial temperatures  $T_0=5$  MeV(left panel) and  $T_0=10$  MeV (right panel). Solid, dashed and dotted lines are the Langevin simulations with constant initial temperature, the Langevin simulation with time dependent temperature and the pure potential calculations, respectively.

the fluctuations of the collective variable Q during the expansion phase are not developed dynamically but originate mainly from the propagation of the initial statistical fluctuations by the driving force. As the system expands, it cools down and consequently the magnitude of the stochastic force in the Langevin equation become smaller. The dashed lines in figures 1 and 2 indicate the simulations carried out by employing a time dependent temperature, which is determined according to  $T(t) = T_0 (R_0/R(t))^2$  by assuming adiabatic expansion, where R(t) represents the root-mean-square radius of the expanding system. As seen from these, the cooling has a minor effect on the probability distribution n(Q,t) during the early phase of the expansion. However, cooling has a sizable effect in momentum space at large times, as seen in the asymptotic value of the  $\sigma_P$  which also come close to the result obtained by the pure potential calculations.

# 3.2 Comparison between quantum and classical dynamics

According to the classical Langevin simulations, during the expansion phase, the fluctuation of collective variable Q is mainly determined by the potential evolution and the friction and stochastic forces have a minor effect. It is reasonable to expect a similar behavior in the quantal evolution of the collective motion. Therefore in the application of the quantal model presented in section 2.3, we neglect the collision term, and calculate the probability distribution n(Q,t) and the variances  $\sigma_Q(t)$ ,  $\sigma_P(t)$  by keeping the occupation probabilities  $\rho_i$  to be constant and equal to initial values specified by the Bolztmann factors. Solid lines in figure 3 illustrates the result of quantal calculations of the probability distribution n(Q,t) at different times for two initial temperatures T = 5 MeV (left panel) and T = 10 MeV (right panel). In the same figure, dotted lines show the classical simulations without friction and stochastic force, and also, as a reference dashed lines show the quantal calculations performed only with the ground state (ie the T=0MeV case). Figure 4 illustrates the evolution of the variances  $\sigma_Q(t)$  and  $\sigma_P(t)$  as a function of time for initial temperatures T=5 MeV and T=10MeV. The classical simulations, the full quantal calculations and the quantal calculations with ground state are shown by dotted, solid and dashed lines, respectively. The fluctuations of the collective variable,  $\sigma_O(t)$ , exhibit an exponential growth as soon as the mode becomes unstable, and saturate at

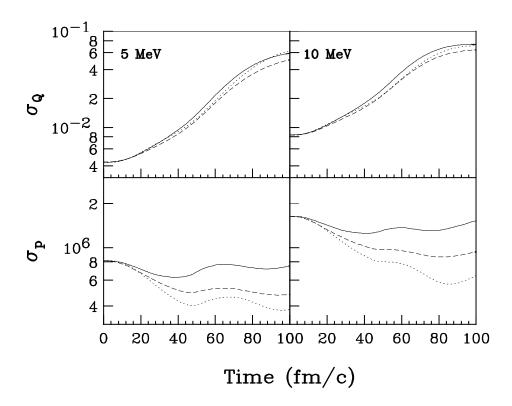


Figure 2: Variances associated with collective variable Q and collective momentum P as a function of time at initial temperatures  $T_0 = 5$  MeV(left panel) and  $T_0 = 10$  MeV (right panel). Solid, dashed and dotted lines are the Langevin simulations with constant initial temperature, the Langevin simulation with time dependent temperature and the pure potential calculations, respectively.

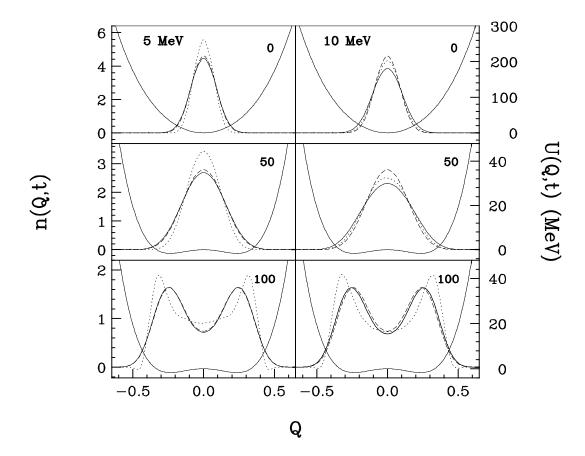


Figure 3: Probability distribution of collective variable and potential energy as a function of Q at times t=0,50,100 fm/c for initial temperatures  $T_0=5$  MeV(left panel) and  $T_0=10$  MeV (right panel). Solid, dashed and dotted lines are the full quantal calculation, the quantal calculations with ground state, and the classical calculations without friction and stochastic forces, respectively.

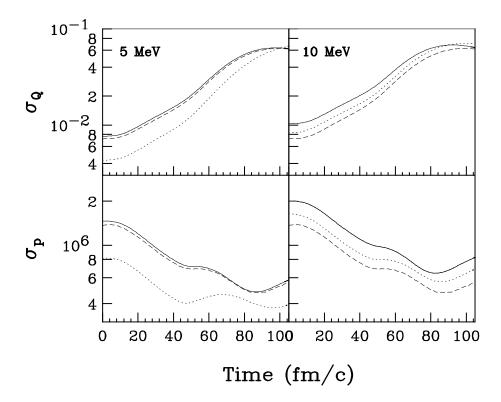


Figure 4: Variances associated with collective variable Q and collective momentum P as a function of time at initial temperatures  $T_0 = 5 \text{ MeV}(\text{left panel})$  and  $T_0 = 10 \text{ MeV}$  (right panel). Solid, dashed and dotted lines are the full quantal calculation, the quantal calculations with ground state, and the classical calculations without friction and stochastic forces, respectively.

large times, in both the classical and quantal calculations. The exponential growth rates are the same in both calculations, but the magnitude of fluctuations are larger in the quantal case. This is illustrated in Figure 5 which shows the critical time  $\tau$  it takes to reach  $\sigma_Q = 0.2$  as a function of the expansion time  $t_0$  of the source for the initial temperatures T = 5 MeV and T = 10 MeV. As seen, the quantal fluctuations shown by dashed lines reduce the time it takes to reach a finite value of the fluctuations. The reduction of this time, depending on the temperature of the system, can be as large as factor of two. For comparison, figure 6 shows different classical calculations of the critical time  $\tau$  as a function of the expansion time  $t_0$ . In this figure, the Langevin simulations with constant initial temperature, the Langevin simulations with time dependent temperature and the pure potential calculations are indicated by solid, dashed and dotted lines, respectively.

## 4 Conclusions

We investigate the development of density fluctuations associated with collective modes (here we specifically consider quadrupole mode) in an expanding nuclear system by employing phenomenological classical and quantal transport equations. According to mean-field calculations, a finite nuclear system prepared at moderate temperatures around T = 5-10 MeV expands until the turning point which is located inside the unstable zone, and then continue to execute monopole vibrations. We calculate the evolution of the probability distribution of the collective coordinate Q and the collective momentum Pand determine the variances associated with these distributions as a function of time. Our investigations indicate that during the expansion phase, the dissipation and fluctuation mechanism play an important role in the evolution of the momentum distribution, but the fluctuations in collective coordinate Q are developed mainly by enhancement of the initial fluctuations of statistical or quantal origin. Furthermore, it appears that quantal fluctuations, which are already present at the initial state, play an important role in particular at low temperatures, in dynamics of symmetry breaking by reducing the time required for reaching the critical fluctuations. Consequently, the spinodal decomposition and the resultant fragmentation may take place in a faster time scale than predicted by semi-classical simulations [6, 19]. Therefore, it is off great interest to develop stochastic simulation methods for heavy-ion collisions by incorporating quantal fluctuations associated with collective motion

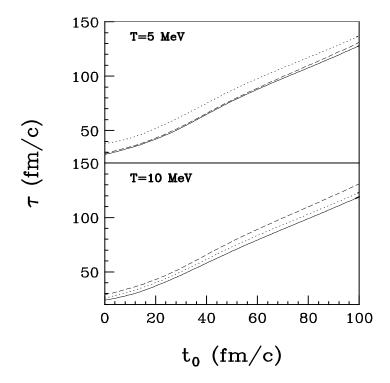


Figure 5: Time  $\tau$  that takes to reach  $\sigma_Q = 0.2$  as a function of the expansion time  $t_0$  at temperatures T = 5 MeV (top panel) and T = 10 MeV (bottom panel). Solid, dashed and dotted lines are the full quantal calculation, the quantal calculations with ground state, and the classical calculations without friction and stochastic forces, respectively.

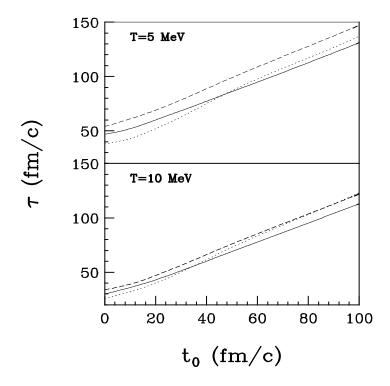


Figure 6: Same as in figure 5. Solid, dashed and dotted lines are the Langevin simulations with constant initial temperature, the Langevin simulation with time dependent temperature and the pure potential calculations, respectively.

in a suitable manner.

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### References

- [1] J. Cugnon, Phys. Lett. **135B** (1986) 374.
- [2] G. Bertsch and P.J. Siemens, Phys. Lett. **126B** (1983) 9.
- [3] S. Ayik and C. Gregoire, Phys. Lett. **B212** (1988) 269; Nucl. Phys. **A513** (1990) 187.
- [4] J. Randrup and B. Remaud, Nucl. Phys. **A514** (1990) 339.
- [5] Y. Abe, S. Ayik, P.G. Reinhard and E. Suraud, Phys. Rep. 275 (1996) 49.
- [6] A. Guarnera Thesis, (1996) G.A.N.I.L Caen, GANIL T.96-01.
   A. Guarnera, M. Colonna and P. Chomaz, Phys. Lett B373, (1996) 267.
- [7] R. Balian and M. Veneroni, Ann. Phys. 135 (1981) 270.
- [8] P.G. Reinhard and E. Suraud, Ann. of Phys. 216 (1992) 98.
- [9] B. Jacquot, M. Colonna, S. Ayik and Ph. Chomaz, Nucl. Phys. A617 (1997) 356.
- [10] S. Ayik et al., Z. Phys. **A337** (1990) 413.
- [11] B. Morgenstern and W. Norenberg, Nucl. Phys. A492 (1989) 93.
- [12] H. Hofmann and P.J. Siemens, Nucl. Phys. A257 (1976) 165; A275 (1977) 464.

- [13] S. Ayik, Z. Phys. A350 (1994) 45; Adv. in Nuclear Dynamics 2, eds.W. Bauer and G. Westfall, Plenum Press, New York (1996).
- [14] S. Ayik and J. Randrup, Phys. Rev. C50 (1994) 2947.
- [15] C.W. Gardiner, Quantum Noise, Springer-Verlag (1991).
- [16] D. Vautherin, J. Treiner and M. Veneroni, Phys. Lett. **B191** (1987) 6.
- [17] D. Lacroix and Ph. Chomaz, preprint GANIL P 98 04 and submitted to Nucl. Phys. A (1998).
- [18] S. E. Koonin, Computational Physics, Addison-Wesley, (1986).
- [19] N. Marie et al., Phys. Lett. B391 (1997) 15.
   M. F. Rivet et al., Proc. XXXV Int. Winter Meeting in Nuclear Physics, ed. I. Iori, Bormio, Italy (1997).